C.1

UNCLASSIFIED RM L51J19

MAR 18 1952

NACA

RESEARCH MEMORANDUM

FOR REFERENCE

NOT TO BE TAKEN FROM THIS ROOM

THEORETICAL CALCULATION OF THE EFFECT OF THE FUSELAGE

ON THE SPANWISE LIFT DISTRIBUTION ON A WING

By Martin Zlotnick and Franklin W. Diederich

Langley Aeronautical Laboratory
Langley Field, Va.

CLASSIFICATION CANCELLED

Author: J.W. Crowley Date 12/14/53
[E.O.10501 OK]
By m.H.A. 1/12/54 See n.u.Sch.y
RF 1828

NATIONAL ADVISORY COMMITTEE
FOR AERONAUTICS

WASHINGTON
March 3, 1952

UNCLASSIFIED

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

RESEARCH MEMORANDUM

THEORETICAL CALCULATION OF THE EFFECT OF THE FUSELAGE

ON THE SPANWISE LIFT DISTRIBUTION ON A WING

By Martin Zlotnick and Franklin W. Diederich

SUMMARY

A method is presented for calculating the effect of the fuselage on the spanwise lift distribution on a wing by an application of a conformal-mapping procedure to the simplified lifting-surface theory. This method is applicable to any symmetrical wing-fuselage configuration. At low angles of attack the method is shown to approach the theoretically correct limit for wing-fuselage configurations having a wing of vanishingly small aspect ratio and for those having a wing of infinite aspect ratio. Some calculations made by this method are compared with experimental data. A computing scheme is given in which the calculation for the infinitely long fuselage and the correction for the finite fuselage length are made separately. A numerical example is given in an appendix to illustrate the entire procedure.

INTRODUCTION

A knowledge of the aerodynamic loading on the wing of an airplane is required for the stress analysis of the wing and for the prediction of the flying qualities of the airplane. Experimental determination of this loading is tedious and is resorted to in very few cases. Satisfactory theoretical methods for the calculation of the spanwise lift distribution on the wing alone have been developed, but the problem of predicting the effect of the fuselage on the spanwise lift distribution on the wing has not yet been satisfactorily treated. Experience has indicated that the effect of the fuselage tends to be small at low angles of attack and, for the lack of any better information, is usually neglected. However, refinements in the methods of stress analysis place a premium on the accurate prediction of the aerodynamic loading applied to the wing and, hence, on the effect of the fuselage on that loading. Also, for swept wings, indications are that the effects of the fuselage on the spanwise lift distribution of the wing are not as small as for unswept wings.

Consequently, development of a theoretical method for predicting these effects is very desirable at this time.

In estimating the effect of the fuselage on the spanwise lift distribution on a wing, it is necessary in general to consider the following:

(1) The fuselage causes a change in the longitudinal velocity in the vicinity of the wing. This effect is hereinafter referred to as the "inflow effect."

(2) If the fuselage is at an angle of attack relative to the free stream, it changes the flow about the wing in planes normal to the free stream. This effect is hereinafter referred to as the "cross flow effect."

(3) The fuselage has a blocking effect, since there can be no flow normal to its surface.

The inflow effect, which is a result of the finite length of the fuselage, changes the longitudinal velocity in the vicinity of the wing and, hence, the lift on the wing. This change is not large for a slender fuselage but may be important when the fuselage is relatively thick, so that a substantial change is caused in the local longitudinal velocity. For the same reason this effect may be important for a wing-nacelle combination.

The cross-flow caused by the fuselage changes the component of free-stream velocity normal to the fuselage axis and also affects the downwash flow produced by the wing. The blocking effect of the fuselage is always present even if the fuselage is an infinite cylinder alined with the free stream, in which case the other two effects are not present.

In the following analysis the blocking and cross-flow effects will be treated by an application of a conformal-mapping procedure to the simplified lifting-surface theory, and the inflow effect will be treated as a separate correction. The conformal-mapping procedure for calculating the blocking and cross-flow effects is derived in a somewhat intuitive fashion from Multhopp's method (reference 1) and is not theoretically justified because it applies a conformal transformation to a flow which is not strictly two-dimensional. However, the method presented herein (referred to as the new method) is shown to be in good agreement with experimental results and to give the same result as more exact theory for two limiting cases - namely, for wing-fuselage configurations having a wing of infinite aspect ratio and for those having a wing of vanishingly small aspect ratio.

SYMBOLS

c	chord
c_{av}	mean chord (S/b)
c_l	section lift coefficient
c_{l_α}	section lift-curve slope (per radian)
α	geometric angle of attack of wing
α_f	geometric angle of attack of fuselage
l	section lift; also length of body
ρ	mass density of air
V	free-stream velocity
U	longitudinal component of free-stream velocity $(V \approx U)$
u	local longitudinal velocity
δ	local velocity increment $\left(\dfrac{u}{V} - 1\right)$
Γ	circulation
y, η	lateral coordinate
x	longitudinal coordinate
z	vertical coordinate
S	wing area
b	wing span
A	aspect ratio $\left(b^2/S\right)$
a	radius of body
h	diameter of body

Λ angle of sweep

λ taper ratio

$w_{3c/4}$ downwash velocity at three-quarter-chord line

w_n component of velocity normal to plane of wing

w_{n_f} component of velocity normal to plane of wing due to
                           fuselage

γ dimensionless circulation $(4\Gamma/bV)$

$\bar{\gamma}$ dimensionless circulation $(4\Gamma/\bar{b}V)$

$\bar{c}*$ chord made dimensionless with respect to transformed semi-
                           span $\left(\dfrac{c}{\bar{b}/2}\right)$

ζ complex coordinate in real plane $(z + iy)$

$\bar{\zeta}$ complex coordinate in transformed plane $(\bar{z} + i\bar{y})$

$R\left(\dfrac{d\bar{\zeta}}{d\zeta}\right)$ real part of conformal-transformation factor

Superscript:

*   indicates coordinate made dimensionless with respect to
                           semispan

Subscripts:

w   refers to wing

wf   refers to wing of the wing-fuselage combination

    A bar indicates transformed coordinate, or quantity made dimension-
less with respect to transformed coordinate.

DESCRIPTION OF THE METHOD

Analysis for the Infinitely Long Fuselage

In order to calculate the fuselage cross-flow and blocking effects on the spanwise lift distribution on a wing, a conformal-mapping procedure is applied herein to the Weissinger L-method (reference 2) in the same manner that Multhopp applies a conformal-mapping procedure to the lifting-line method.

Inasmuch as the concept of conformal mapping is based on two-dimensional considerations and inasmuch as the simplified lifting-surface method cannot, in general, be reduced to a two-dimensional problem as can lifting-line theory, the method outlined herein is not rigorously justifiable. However, for wings of very low aspect ratio, this method is found to be justifiable because in those cases the simplified lifting-surface method can be reduced to a two-dimensional problem. The results obtainable by the method under consideration will also be shown to constitute a very close approximation to the theoretically correct results in the case of wings of infinite aspect ratio. Therefore, there is reason to believe that this method will furnish useful answers for wings of intermediate aspect ratios.

The integrodifferential equation of the simplified lifting-surface method is given in reference 2. In the presence of the fuselage the normal-flow component on the wing w_n is

$$w_n = V\alpha + w_{n_f} \qquad\qquad (1)$$

where α is the local angle of attack on the wing and w_{n_f} is the incremental normal flow on the wing caused by the fuselage. The lift distribution is now calculated for a wing-fuselage configuration obtained from the given one by a conformal transformation (in planes normal to the free-stream velocity) in which the cross section of the fuselage is a vertical slit, so that there is no fuselage interference. The velocity components normal to the free-stream velocity and parallel to the plane of symmetry are then multiplied by the conformal-transformation factors and the simplified lifting-surface equation is written for the wing in physical space,

but in terms of the transformed coordinates. Thus, in physical space,
the normal-flow component caused by the fuselage is

$$w_{n_f} = V\alpha_f(\overline{y})R\!\left(\frac{d\overline{\zeta}}{d\zeta}\right) - V\alpha_f(\overline{y}) \tag{2}$$

where $\alpha_f(\overline{y})$ is the angle of attack of the fuselage; also, the downwash
velocity at the three-quarter-chord line becomes

$$w_{3c/4} = R\!\left(\frac{d\overline{\zeta}}{d\zeta}\right)\overline{w}_{3c/4}(\overline{y}) \tag{3}$$

where $R\!\left(\frac{d\overline{\zeta}}{d\zeta}\right)$ is the real part of the conformal-transformation factor as
in Multhopp's treatment of the wing-fuselage combination. As a result of
the preceding relations the integrodifferential equation expressing the
lift in terms of the wing and fuselage geometry becomes

$$\alpha(\overline{y}^*) + \alpha_f(\overline{y}^*)\left[R\!\left(\frac{d\overline{\zeta}}{d\zeta}\right) - 1\right] = R\!\left(\frac{d\overline{\zeta}}{d\zeta}\right)\left[\frac{1}{4\pi}\int_{-1}^{1}\frac{d\overline{\gamma}}{d\overline{\eta}^*}\,\frac{d\overline{\eta}^*}{\overline{y}^* - \overline{\eta}^*} + \right.$$

$$\left. \frac{1}{8\pi\frac{\overline{c}^*}{2}}\int_{-1}^{1}L(\overline{y}^*,\overline{\eta}^*)\frac{\partial\overline{\gamma}}{\partial\overline{\eta}^*}\,d\overline{\eta}^*\right] \tag{4}$$

$$\frac{\alpha(\bar{y}*) - \alpha_f(\bar{y}*)}{R\left(\frac{d\bar{\zeta}}{d\zeta}\right)} + \alpha_f(\bar{y}*) = \frac{1}{4\pi} \int_{-1}^{1} \frac{d\bar{\gamma}}{d\bar{\eta}*} \frac{d\bar{\eta}*}{\bar{y}* - \bar{\eta}*} +$$

$$\frac{1}{8\pi \frac{\bar{c}*}{2}} \int_{-1}^{1} L(\bar{y}*,\bar{\eta}*)\frac{\partial\bar{\gamma}}{\partial\bar{\eta}*} \, d\bar{\eta}* \qquad (5)$$

Comparison of equation (5) with equation (34) of reference 2 shows that the procedure for calculating the lift distribution on the wing-fuselage combination is only a slightly modified form of that required for the wing alone.

### Computing Procedure for a Midwing Configuration

In order to illustrate the method, the case of a cylindrical fuselage with a midwing is outlined in this section and a numerical example for a configuration with a "shoulder high" wing is given in the appendix. Other fuselage cross sections and wing locations may be treated in the same manner provided the correct transformation factor is used. (See reference 1.)

The lateral and vertical coordinates in physical space, y and z, respectively, are combined into one complex coordinate

$$\zeta = z + iy \qquad (6)$$

Similarly, these coordinates in the space obtained by transforming each plane in physical space normal to the free-stream velocity are combined into a complex coordinate
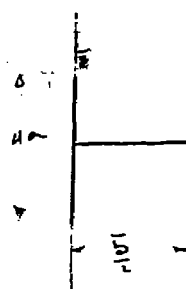
$$\bar{\zeta} = \bar{z} + i\bar{y} \qquad (7)$$
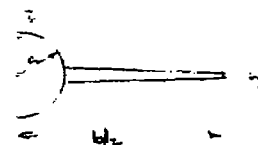
For a cylindrical fuselage, the relation between the two complex coordinates is

$$\bar{\zeta} = \zeta + \frac{a^2}{\zeta} \qquad (8)$$

where a is the radius of the fuselage. The calculation for the spanwise lift distribution is then made as follows:

(1) In the transformed plane, the coordinate $\overline{y}$ for a midwing configuration is

$$\overline{y} = y\left(1 - \frac{a^2}{y^2}\right) \tag{9}$$

so that when $y = a$, $\overline{y} = 0$. When $y = \frac{b}{2}$, $\overline{y} = \frac{\overline{b}}{2}$, so that

$$\frac{\overline{b}}{b} = 1 - \left(\frac{a}{b/2}\right)^2 \tag{9a}$$

Also, since the longitudinal coordinate is not changed by the transformation,

$$\overline{x} = x \tag{9b}$$

(2) Since

$$\frac{d\overline{\zeta}}{d\zeta} = 1 - \frac{a^2}{\zeta^2} \tag{10}$$

the real part of the conformal-transformation factor is

$$R\left(\frac{d\overline{\zeta}}{d\zeta}\right) = 1 + \frac{a^2}{y^2} \tag{11}$$

for $z = 0$, the vertical position of the wing.

(3) If the wing is unswept, equation (5) is solved in the usual manner for the transformed wing, except that $\alpha - \alpha_f$ is divided by $R\left(\frac{d\overline{\zeta}}{d\zeta}\right)$. The

difference $\alpha - \alpha_f$ is 0 when the wing is not set at an angle of incidence with the fuselage axis.

(4) If the wing is swept, another method must be used for treating the transformed wing, since the transformed swept wing has a curved quarter-chord line (see fig. 1), a case for which Weissinger's method cannot be applied.

The simplest procedure in this case is probably the use of horseshoe vortices (about 10 centered on the quarter-chord line on each semispan appear to be a satisfactory number). By calculating the downwash induced by each vortex at 10 points on the three-quarter-chord line of the semispan and equating the sum to the slope of the wing at the three-quarter-chord line, a set of 10 simultaneous equations is obtained for the 10 unknown strengths of the vortices. (See the appendix.) In calculating the downwash due to the vortices, the tables in reference 3 are very helpful.

(5) The relation between the dimensionless circulation on the wings in physical and transformed space is

$$\gamma = \frac{\overline{b}}{b} \, \overline{\gamma} \tag{12}$$

(6) The dimensionless circulation γ is plotted at the lateral coordinate in the physical plane which, from step (2), is

$$y^* = \frac{\overline{b}}{b} \frac{\overline{y}^* + \sqrt{\overline{y}^{*2} + 4\left(\frac{b}{\overline{b}} \, a^*\right)^2}}{2} \tag{13}$$

where

$$\overline{y}^* = \frac{\overline{y}}{\overline{b}/2}$$

If, for instance, the lift distribution is known on a rectangular wing alone, the lift distribution for the combination of this wing and a fuselage can be obtained quite readily. The results of the calculation for the wing alone can be assumed to be the results for the transformed

wing except for a small correction which can be made by applying lifting-line theory. This procedure may be used if the ratio of the span to the fuselage diameter is large—so that $\bar{b}/b$ (equation (9a)) is nearly unity. The transformation is then made back into the physical plane so that the lift is plotted as indicated in step (6). If γ_{wf} is the dimensionless circulation on the wing-fuselage combination and $\bar{\gamma}_w$ is the dimensionless circulation on the wing alone, then

$$\gamma_{wf}(y^*) = \left[\bar{\gamma}_w(\bar{y}^*)\frac{\bar{b}}{b}\right]\frac{\frac{b}{\bar{b}}A + 2}{A + 2} \tag{14}$$

The correction applied in equation (14) is based on the lifting-line solution for the elliptical wing

$$c_l = 2\pi \frac{A}{A + 2}\alpha \tag{15}$$

and the identity which holds in the case of a rectangular wing

$$\gamma = \frac{2}{A}c_l \tag{16}$$

Correction for the Finite Length of the Fuselage

In representing the fuselage by an infinite cylinder the inflow effect is neglected. The lateral distribution of the induced velocity is shown in figure 2 for two fairly typical bodies; the body with the slenderness ratio of 3 approximates a nacelle and the body with the slenderness ratio of 7 approximates a slender fuselage. An increase of the effective free-stream dynamic pressure is the immediate result of the induced longitudinal velocity. If the increase in the velocity is small and does not have a large lateral gradient (as, for example, in the case of the slender body in fig. 2) and the wing has a high aspect ratio, the effect on the spanwise lift distribution may be calculated as follows:

If the local velocity at a given spanwise station is $U(1 + \delta)$ where $\delta \ll 1$, the lift in the region of increased velocity is

$$l = c_l c \frac{1}{2}\rho U^2(1 + \delta)^2 \tag{17}$$

or

$$l \approx c_l c \frac{1}{2} \rho U^2 (1 + 2\delta) \tag{18}$$

but the lift in the uniform flow l_o is

$$l_o = c_l c \frac{1}{2} \rho U^2 \tag{19}$$

therefore,

$$l = (1 + 2\delta) l_o \tag{20}$$

Consequently, the lift calculated by the method outlined in the preceding section can be corrected approximately for the inflow effect by multiplying it by the factor $1 + 2\delta$ appropriate to the given fuselage and any spanwise station.

It should be noted that the value of δ on the surface of a given fuselage varies with the longitudinal position of the wing and also with the vertical location of the wing when the fuselage is at an angle of attack. From figure 10 of reference 4, more lift is seen to be expected on a high-wing than on a low-wing configuration if the wing is located ahead of the position of the fuselage maximum diameter. It should also be noted that, since the wing is of finite thickness, the longitudinal velocity induced by the fuselage may be different on the wing upper surface from that on the lower surface; this difference tends to increase or decrease the circulation on the wing and, hence, to change its lift accordingly. The inflow correction given before may therefore be applied only when the wing section is thin relative to the fuselage diameter.

DISCUSSION OF METHOD AND CALCULATED RESULTS

Comparison of the Results of the New Method for the Limiting Case of

Vanishingly Small Aspect Ratio with the Results of

Linearized Low-Aspect-Ratio Theory

In order to obtain the spanwise lift distribution on the wing-fuselage combination for the limiting case of vanishingly small aspect ratio, it is only necessary to multiply the value of $\gamma(y^*)$, the loading coefficient for the wing alone, by the factor $\bar{b}/b$ and to plot the result γ_{wf} at the points on the wing-fuselage in the physical plane which correspond to the points for the wing alone in the transformed plane. (See steps (5) and (6) of the section entitled "Computing Procedure for a Midwing Configuration.") The relation between the lateral coordinates in the physical plane and the lateral coordinates in the transformed plane is given in equation (9). The solution for the wing alone of vanishingly small aspect ratio may be obtained from reference 5 as

$$\bar{\gamma}_w(\bar{y}^*) = 4\sqrt{1 - \bar{y}^{*2}} \tag{21}$$

which can be used for the wing-fuselage combination by setting

$$\gamma_{wf}(y^*) = \frac{\bar{b}}{b}\,\bar{\gamma}_w(\bar{y}^*) \tag{22}$$

Figure 3 shows a comparison of the results obtained by using the new method for the limiting case of vanishingly small aspect ratio with the results of Spreiter's method (reference 6) which is theoretically correct for the limiting case. As would be expected, no discrepancy is apparent because, in this limiting case, the problem of calculating the lift distribution is essentially a two-dimensional one (in planes perpendicular to the stream); thus, the procedure of transforming the physical space plane for plane into a distorted space is rigorously justifiable.

It should be noted that the results shown in figure 3 apply only to configuration with wings of very low aspect ratio (A < 1) where the flow is essentially two-dimensional in planes normal to the free stream and, therefore, the trend of decreased lift on the wing in the presence of the fuselage would not necessarily apply to configurations with wings of moderate or high aspect ratio.

Comparison of the Results of the New Method at the Limiting Case of

Infinite Aspect Ratio with the Results Obtained from

Considerations of an Idealized Case

The calculation made by Lennertz (reference 7) to determine the span-
wise lift distribution on a combination of a constant-strength vortex
with a cylinder alined with the free stream can be modified slightly to
give the lift distribution when the cylinder axis is placed at an angle
of attack with the free stream. In this case a vertical velocity com-
ponent w_n normal to the axis of the cylinder must be considered in
addition to the longitudinal velocity U considered by Lennertz so that
the resultant stream velocity V is

$$V = \sqrt{w_n^2 + U^2} \qquad (23)$$

and the angle of attack of the cylinder is

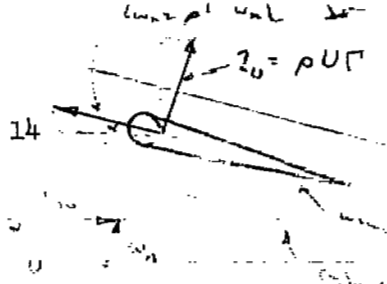$$\alpha_{cyl} = \tan^{-1} \frac{w_n}{U} \qquad (24)$$

In a potential flow this additional upward velocity cannot change the
lift on the cylinder; however, this velocity can give rise to an additional
lift on the part of the wing (represented by the vortex) adjacent to the
fuselage.

The magnitude of this increase in the lift will be calculated by
treating the components of the lift l_U and l_{w_n}, due, respectively, to
the components of the free-stream velocity U and w_n, as follows:

The component of the lift l_U on the wing which acts normal to the
cylinder axis is

$$l_U = \rho U \Gamma \qquad (25)$$

The component of the lift l_{w_n} which acts parallel to the cylinder axis
and opposite in sense to the velocity U is

$$l_{w_n} = \rho \left[w_n \left(1 + \frac{a^2}{y^2} \right) \right] \Gamma \tag{26}$$

where $w_n \left(1 + \frac{a^2}{y^2} \right)$ is the velocity of the vertical flow at the vortex line in the presence of the cylinder. But, as a result of equation (24),

$$l_{w_n} = \rho U \Gamma \tan \alpha \left(1 + \frac{a^2}{y^2} \right) \tag{27}$$

The component of l_U normal to the free-stream velocity is

$$l_U{}' = l_U \cos \alpha \tag{28}$$

and the component of l_{w_n} normal to the free-stream velocity is

$$l_{w_n}{}' = l_{w_n} \sin \alpha \tag{29}$$

The resultant lift l normal to the free-stream velocity is therefore

$$l = l_U \cos \alpha + l_{w_n} \sin \alpha$$

$$= \rho U \Gamma \cos \alpha + \rho U \Gamma \frac{\sin^2 \alpha}{\cos \alpha} \left(1 + \frac{a^2}{y^2} \right)$$

$$= \frac{\rho U \Gamma}{\cos \alpha} \left(1 + \frac{a^2}{y^2} \sin^2 \alpha \right)$$

$$= \rho V \Gamma \left(1 + \frac{a^2}{y^2} \sin^2 \alpha \right) \tag{30}$$

since $U = V \cos \alpha$. Now, if the angle of attack α is small, the term $\frac{a^2}{y^2} \sin^2\alpha$ is negligible; therefore

$$l \approx \rho V \Gamma \tag{31}$$

which is the result for the wing alone. Consequently, as a first-order approximation, the lift distribution on the unswept wing of infinite aspect ratio is unaffected by the presence of the fuselage. The same result is given by equation (14) for the limiting case of the wing-fuselage combination having a wing of infinite aspect ratio.

Comparison of the Calculations with Experiment

The results of calculations made by the method presented in this paper are compared in figures 4 and 5 with experimental results obtained from references 8, 9, 10, and from unpublished data. In figure 6 the results of a calculation by Multhopp's method are shown and compared with experimental data from references 8 and 9. For the unswept wing (figs. 4 and 6), the fuselage was assumed to be a Rankine solid $\frac{l}{h} = 7.0$ (fig. 2) and for the swept wing the fuselage was assumed to be an ellipsoid of revolution $\frac{l}{h} = 10.0$.

Experimental and theoretical results shown in figures 4 and 5 indicate that the swept wing is affected more by the fuselage attached to it than is the unswept wing, although the fuselage radius of the swept wing configuration is only $0.1\frac{b}{2}$ as compared with a fuselage radius of $0.19\frac{b}{2}$ on the unswept wing configuration. The theoretically calculated increase in lift is attributed largely to the decreased sweep at the root of the transformed wing (fig. 1).

It should be noted that, in calculating the inflow effect, the wing thickness was assumed to be negligible in each case, although the wing thickness was 10 percent of the fuselage diameter and 15 percent of the fuselage diameter for the unswept and swept configurations, respectively. This assumption leads to a value for the lift which is slightly less than the correct value.

Figures 4 and 6 show comparisons of experimental results of references 8 and 9 with results of calculations made by the method of this paper and the method of Multhopp. A large discrepancy exists at the

root of the wing between Multhopp's results and the results of experiment, whereas the agreement of experimental results with the results calculated by the method of this paper is fairly good along the entire span. The large lift calculated by Multhopp at the root of the wing is attributed by him to his neglect of the effect of the fuselage on the downwash of the bound vortex, since he calculates the downwash infinitely far behind the wing rather than at the three-quarter-chord line.

CONCLUDING REMARKS

A method has been presented for calculating the lift distributions in incompressible flow over swept or unswept wings with fuselages. This method is more generally applicable than methods presented heretofore and the calculations made by this method are in fair agreement with experimental results. Although further comparison with experimental lift distributions is desirable before the method is applied generally, one conclusion may be drawn from the available results: The presence of a slender fuselage does not have an important effect on the lift distribution on an unswept wing of moderate aspect ratio, but a larger change in the lift distribution on a wing in the presence of a fuselage may be anticipated if the wing is swept.

Langley Aeronautical Laboratory
    National Advisory Committee for Aeronautics
        Langley Field, Va.

APPENDIX

ILLUSTRATIVE EXAMPLE OF THE CALCULATION OF THE SPANWISE LIFT

DISTRIBUTION ON THE WING IN THE PRESENCE OF THE FUSELAGE

Geometry of the Wing-Fuselage Combination

A plan view and elevation of the wing-fuselage combination considered in the illustrative example are shown in figure 7. Some of the pertinent geometrical parameters are:

$$A = 8.02 \qquad a = 0.10\frac{b}{2}$$

$$\lambda = 0.45 \qquad \frac{l}{h} = 10$$

$$\Lambda = 45^{\circ} \qquad z_F = 0.05\frac{b}{2}$$

Calculation of Coordinates of Transformed Quarter-Chord Line

The coordinates of the wing are transformed according to the formulas (see equation (8) of text and reference 1)

$$\overline{y} = y\left(1 - \frac{a^2}{y^2 + z_F{}^2}\right)$$

$$\frac{\overline{y}}{\overline{b}/2} = \frac{b}{\overline{b}}y^*\left(1 - \frac{a^{*2}}{y^{*2} + z_F{}^{*2}}\right)$$

where $y = \frac{b}{2}$, $\overline{y} = \frac{\overline{b}}{2}$, and $\frac{\overline{b}}{b} = 1 - \frac{a^{*2}}{1 + z_F{}^{*2}}$.

At a given value of y^*

$$\frac{x}{b/2} = \frac{\overline{x}}{\overline{b}/2}$$

The coordinates of the quarter-chord lines of the transformed and untransformed wing are as follows:

y^*	$\dfrac{\overline{y}}{\overline{b}/2}$	$x^* = \dfrac{\overline{x}}{\overline{b}/2}$
0.08667 (wing root)	0	0
.1	.020	.0133
.2	.154	.1133
.3	.270	.2133
.4	.379	.3133
.5	.484	.4133
.6	.588	.5133
.7	.692	.6133
.8	.795	.7133
.9	.898	.8133
1.0	1.000	.9133

Calculation of Chords on the Transformed Wing

The formula for the chord length at a given spanwise station may be written

$$c^* = \frac{4}{A(1 + \lambda)}\left[1 - (1 - \lambda)y^*\right]$$

therefore

$$\frac{c}{\overline{b}/2} = \frac{b}{\overline{b}}c^* = \frac{b}{\overline{b}}\frac{4}{A(1 + \lambda)}\left[1 - (1 - \lambda)y^*\right]$$

For the plan form being treated

$$\frac{c}{\overline{b}/2} = \frac{1.01 \times 4}{8.02 \times 1.45}\left(1 - 0.55y^*\right)$$

The chords may be tabulated as follows:

$\dfrac{\overline{y}}{b/2}$	y^*	$\dfrac{c}{b/2}$
0	0.0866	0.331
.05	.120	.325
.15	.198	.310
.25	.283	.293
.35	.374	.276
.45	.468	.258
.55	.561	.240
.65	.660	.221
.75	.758	.203
.85	.854	.184
.95	.951	.166

Calculation of Lift Distribution on the Transformed Wing

The lift on the wing is calculated by replacing the wing by a set of 20 lifting horseshoe vortices located on the quarter-chord line at the spanwise stations $y^* = \pm 0.05, \pm 0.15, \pm 0.25, \pm 0.35, \pm 0.45, \pm 0.55, \pm 0.65, \pm 0.75, \pm 0.85,$ and ± 0.95. The downwash induced by these vortices on the three-quarter-chord line of the wing at these same spanwise stations is equated to the angle of attack of the wing at the three-quarter-chord line. This procedure yields a set of 10 simultaneous equations of the form (reference 3):

$$\alpha_{3c/4}(\overline{x}^*,\overline{y}^*) = 45.60 \sum_{n=1}^{20} \left(\frac{cc_{l_\alpha}}{b/2}\right)_n F_n\left[\left(\overline{x}^* - \overline{x}_n^*\right), \left(\overline{y}^* - \overline{y}_n^*\right)\right]$$

The numbers are arranged so that

$$\overline{y}_n^* = \frac{n - \frac{1}{2}}{10} \qquad (1 \leqq n \leqq 10)$$

and

$$\overline{y}_n^* = -y^*_{n-10} \qquad (11 \leqq n \leqq 20)$$

The longitudinal location of the quarter-chord line of the wing is represented by $\bar{x}_n{}^*$ which corresponds to $\bar{y}_n{}^*$ and $\left(\dfrac{cc_{l_\alpha}}{\overline{b}/2}\right)_n$ is the loading coefficient of station $\bar{x}_n{}^*, \bar{y}_n{}^*$.

Since the load is symmetrical,

$$\left(\frac{cc_{l_\alpha}}{\overline{b}/2}\right)_n = \left(\frac{cc_{l_\alpha}}{\overline{b}/2}\right)_{n-10} \qquad (11 \leqq n \leqq 20)$$

and the equations may therefore be written

$$\alpha_{3c/4}(\bar{x}^*,\bar{y}^*) = 45.60\sum_{m=1}^{10} \left(\frac{cc_{l_\alpha}}{\overline{b}/2}\right)_m G_m\left[(\bar{x}^* - \bar{x}_m{}^*), (\bar{y}^* - \bar{y}_m{}^*)\right] \qquad (A1)$$

where

$$\left(\frac{cc_{l_\alpha}}{\overline{b}/2}\right)_m = \left(\frac{cc_{l_\alpha}}{\overline{b}/2}\right)_n \qquad (1 \leqq n \leqq 10)$$

and

$$G_m = F_n + F_{n+10} \qquad (1 \leqq n \leqq 10)$$

The values of F may be most conveniently obtained from the tables in reference 3. In the notation of reference 3,

$$\Delta x_v = 20\left(\bar{x}^* - \bar{x}_m{}^*\right)$$

$$\Delta y_v = 20\left(\bar{y}^* - \bar{y}_m{}^*\right)$$

Equation (A1) may be written for the 10 different values of $\bar{y}^*$. Since the values of α and F are known in each of the 10 equations, the equations may be solved simultaneously for the values of $\dfrac{cc_{l_\alpha}}{\overline{b}/2}$.

The values of $\dfrac{cc_{l_\alpha}}{\overline{b}/2}$ so obtained are given in the following table and are multiplied by $\overline{b}/b$ to give $\dfrac{cc_{l_\alpha}}{b/2}$:

$\dfrac{\overline{y}}{b/2}$	y^*	$\dfrac{cc_{l_\alpha}}{\overline{b}/2}$	$\dfrac{cc_{l_\alpha}}{b/2}$
0.05	0.120	1.085	1.074
.15	.198	1.087	1.077
.25	.283	1.078	1.067
.35	.374	1.046	1.036
.45	.468	1.000	.990
.55	.561	.947	.938
.65	.660	.895	.886
.75	.758	.832	.824
.85	.854	.752	.744
.95	.951	.622	.616

Calculation of the Inflow Effect

The value of u/U on the surface of a body of revolution is obtained very simply from equation (22) of reference 4. For an ellipsoid of revolution, this equation reduces to

$$\frac{u}{U} = \left[\frac{1 - \left(\frac{2x}{l}\right)^2}{v^2 - \left(\frac{2x}{l}\right)^2}\right]^{1/2} \left[\left(\frac{h}{l}\right)^2 Q_1(v) + v\right]$$

where

$$v = \sqrt{1 + \left(\frac{h}{l}\right)^2}$$

and

$$Q_1(v) = \frac{v}{2} \log_e\left(\frac{v+1}{v-1}\right) - 1$$

When l/h is 10 and x is 0, then $v = 1.005$ and

$$Q_1 = 0.5025 \log_e \frac{2.005}{0.005} - 1 = 2.01$$

and

$$\frac{u}{U} = \frac{1}{1.005} (0.01 \times 2.01 + 1.005) = 1.02$$

But, since $\frac{u}{U} = 1 + \delta$, then δ must be 0.02 on the surface of the body. Since δ is small, the lateral variation of δ can be taken to be the same as the lateral variation of δ in figure 2 without any significant loss in accuracy in the final result.

As shown in the text, the lift at a section is multiplied by the value of $1 + 2\delta$ which corresponds to the spanwise location of the section to correct for the inflow effect. The values of $1 + 2\delta$ and of the corrected lift distribution in the case of this example are as follows:

y^*	$1 + 2\delta$	$\dfrac{cc_{l_\alpha}}{b/2}$ (corrected for inflow effect)
0.120	1.04	1.117
.198	1.04—	1.120
.283	1.04	1.110
.374	1.03	1.067
.468	1.03	1.020
.561	1.02	.956
.660	1.02	.904
.758	1.01	.832
.854	1.01	.752
.951	1.00	.616

This lift distribution is plotted in figure 5.

REFERENCES

1. Multhopp, H.: Aerodynamics of the Fuselage. NACA TM 1036, 1942.

2. Weissinger, J.: The Lift Distribution of Swept-Back Wings. NACA TM 1120, 1947.

3. Diederich, Franklin W.: Charts and Tables for Use in Calculations of Downwash of Wings of Arbitrary Plan Form. NACA TN 2353, 1951.

4. Young, A. D., and Owen, P. R.: A Simplified Theory for Streamline Bodies of Revolution, and Its Application to the Development of High-Speed Low-Drag Shapes. R. & M. No. 2071, British A.R.C., 1943.

5. Diederich, Franklin W., and Zlotnick, Martin: Theoretical Spanwise Lift Distributions of Low-Aspect-Ratio Wings at Speeds below and above the Speed of Sound. NACA TN 1973, 1949.

6. Spreiter, John R.: Aerodynamic Properties of Slender Wing-Body Combinations at Subsonic, Transonic, and Supersonic Speeds. NACA TN 1662, 1948.

7. Lennertz, J.: Beitrag zur theoretischen Behandlung des gegenseitigen Einflusses von Tragfläche und Rumpf. Z.f.a.M.M., Bd. 7, Aug. 1927, pp. 249-276.

8. Holme, O.: Comparative Wind Tunnel Tests of a Swept-Back and a Straight Wing Having Equal Aspect Ratios. Rep. No. 31, Aero. Res. Inst. of Sweden (Stockholm), 1950.

9. Jacobs, Willi: Lift and Moment Changes Due to the Fuselage for a Yawed Aeroplane with Unswept and Swept Wings. Rep. No. 34, Aero. Res. Inst. of Sweden (Stockholm), 1950.

10. Graham, Robert R.: Low-Speed Characteristics of a 45° Sweptback Wing of Aspect Ratio 8 from Pressure Distributions and Force Tests at Reynolds Numbers from 1,500,000 to 4,800,000. NACA RM L51H13, 1951.
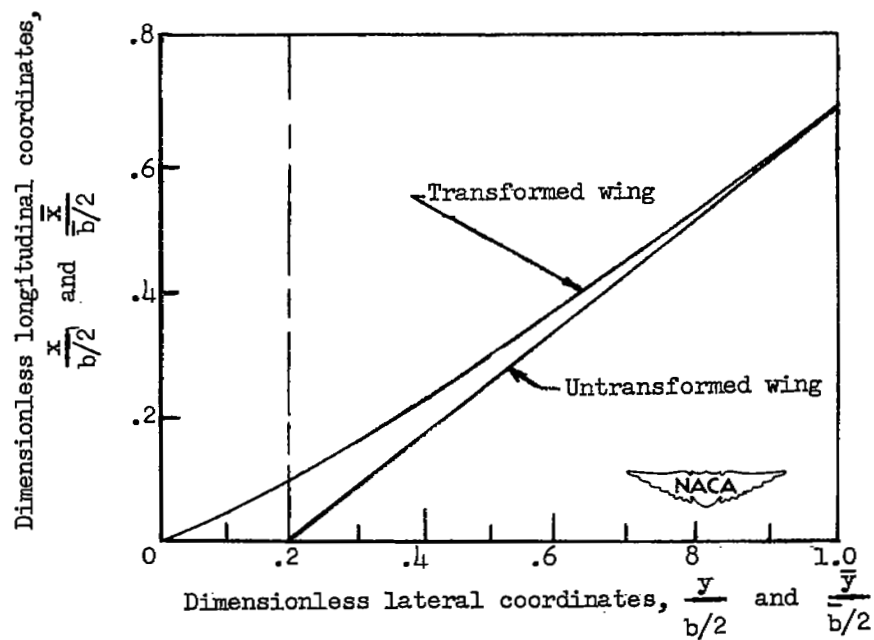
Figure 1.- Dimensionless coordinates of quarter-chord lines of transformed and untransformed swept wings.
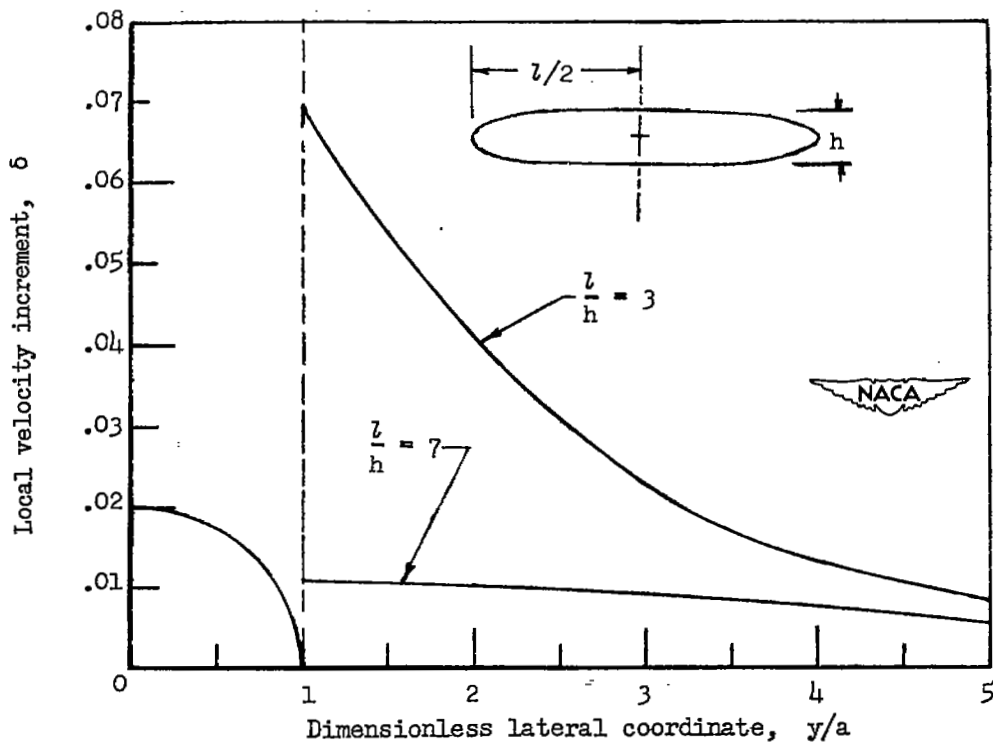

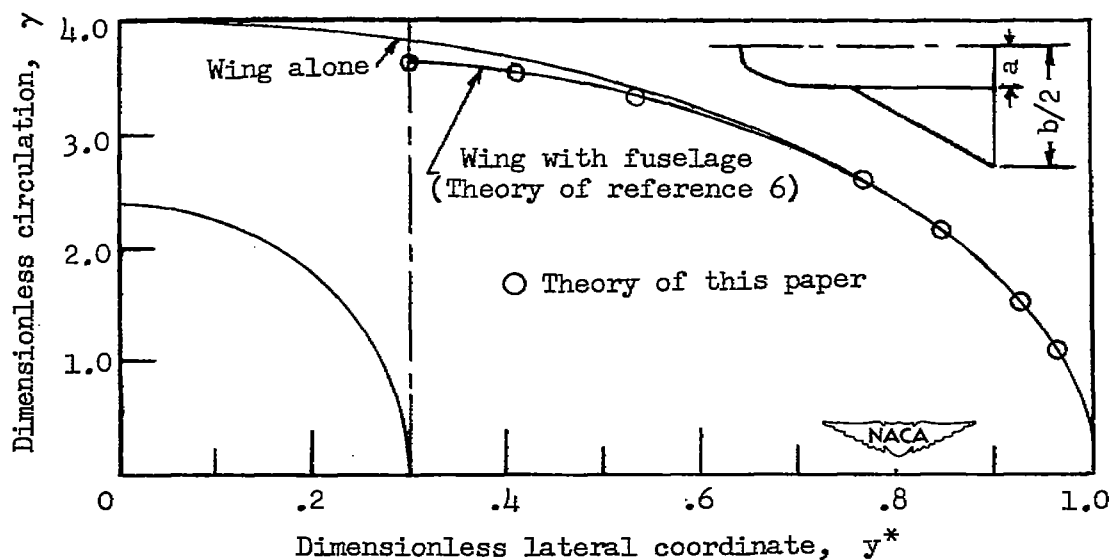Figure 2.- Lateral velocity distribution for two Rankine solids.

Figure 3.- Theoretical lift distribution on a combination of a fuselage
with a wing of low aspect ratio.   a* = 0.3.
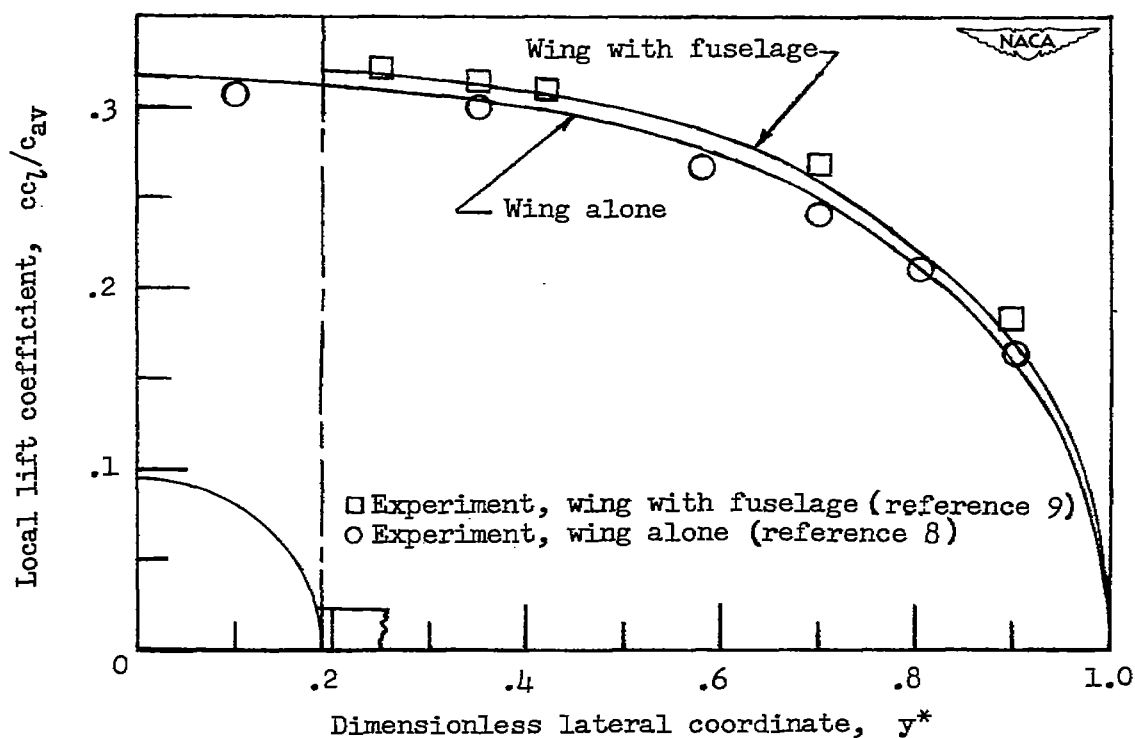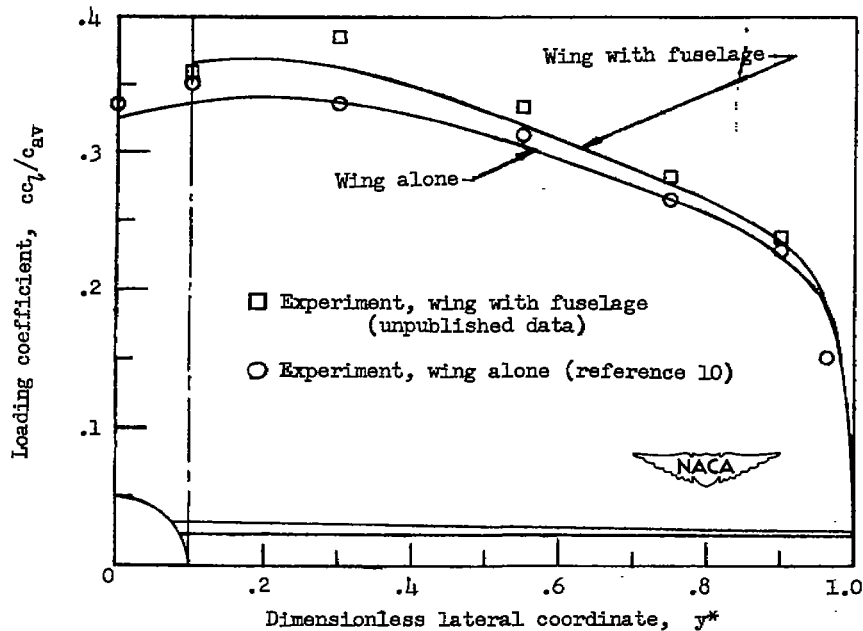

Figure 4.- Comparison of theoretical and experimental lift distributions
on unswept wing with and without fuselage. A = 4.5; λ = 1; Λ = 0°;
a* = 0.19; α = 4°.

Figure 5.- Comparison of theoretical and experimental lift distribution on a swept wing with and without fuselage. $A = 8$; $\lambda = 0.45$; $\Lambda = 45°$; $a* = 0.1$; $\alpha = 4.7°$.

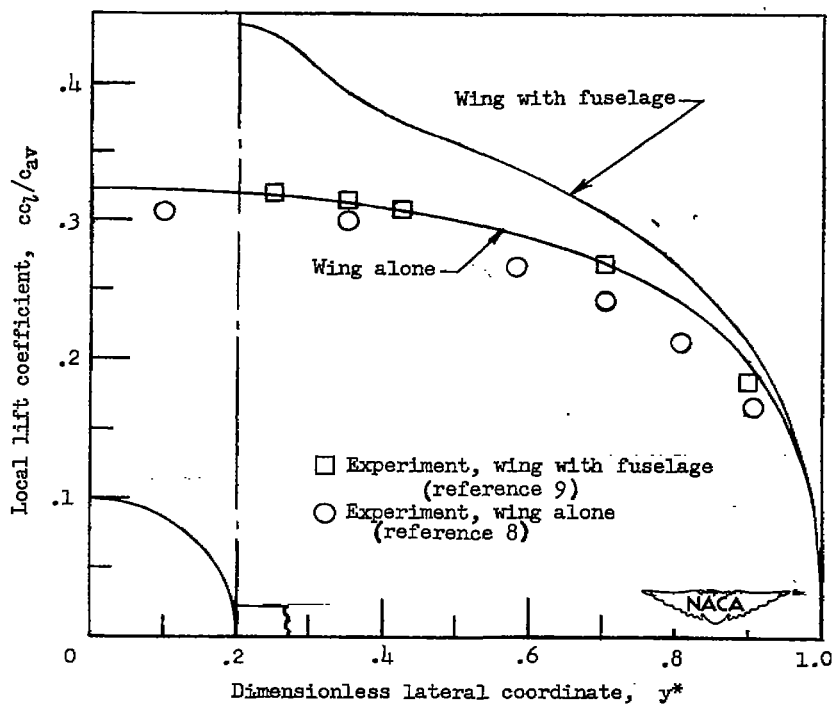
Figure 6.- Calculation for wing with and without fuselage by Multhopp's method. $A = 4.5$; $\lambda = 1$; $\Lambda = 0°$; $a* = 0.2$; $\alpha = 4°$.

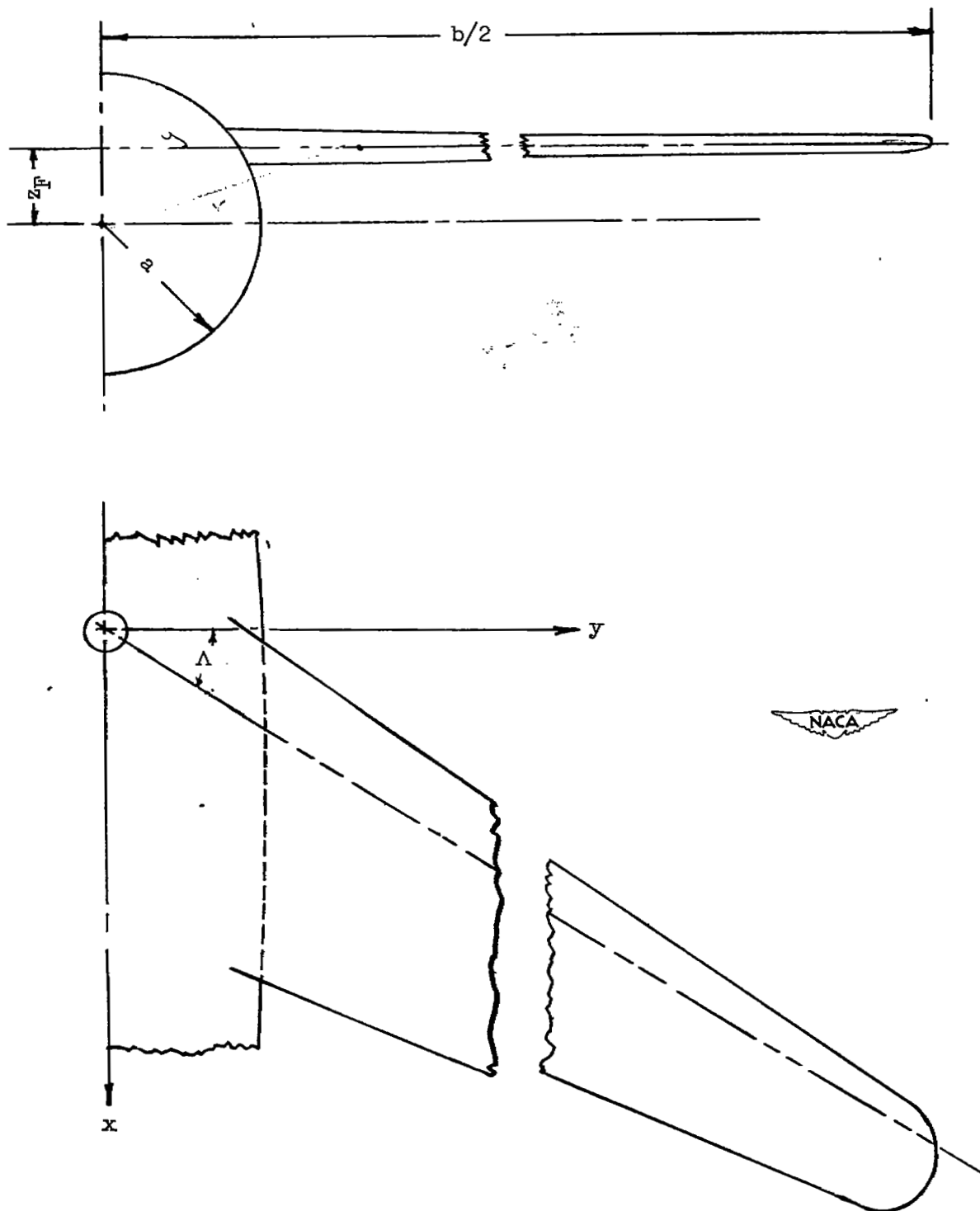Figure 7.- Plan view and elevation of wing-fuselage configuration
considered in illustrative example.